

**Output Voltage Ripple Analysis for  
DCM of Buck, Boost, and Buck-Boost  
Switching Power Supplies**

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04/21/99  
Term Paper Submission

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Power Electronics II  
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Spring 1999

# **Output Voltage Ripple Analysis for DCM of Buck, Boost, and Buck-Boost Switching Power Supplies**

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## **Introduction**

Moving from Power Electronics I to Power Electronics II was a giant step into a mathematical abyss. Subject material is far more difficult to understand, and the related analytical mathematics is extremely time consuming and prone to errors. Few references exist on most of this material, and troubleshooting problems encountered while performing mathematical analysis is consequently very difficult.

Dealing with the discontinuous conduction mode (DCM) of even the most basic types of switching power supplies was very frustrating. Specifically, the idea behind analyzing the output voltage ripple of these converters in the DCM mode should result in the continuous conduction mode result if the proper parameters are set correctly. This paper will spell out the difficulties encountered with this approach, and give a detailed mathematical explanation of the output voltage ripple for three types of converters. The finished product will almost read like an addition to chapter 7 of the text for this class, which was written by the instructor, Dr. Batarseh. Some knowledge of basic switching power supplies is required.

The beginning of the paper will review the CCM output voltage ripple analysis of the buck converter, and then move on to the same analysis for the DCM case. Basic assumption of the CCM mode will be made for the boost and buck-boost converters. The DCM analysis will be given in detail. All results will be compared to the CCM mode, and some hints will be given to aid in comparing CCM mode to DCM mode in these and other similar converters that could save vast amounts of time to someone performing voltage or current analysis on like converters.

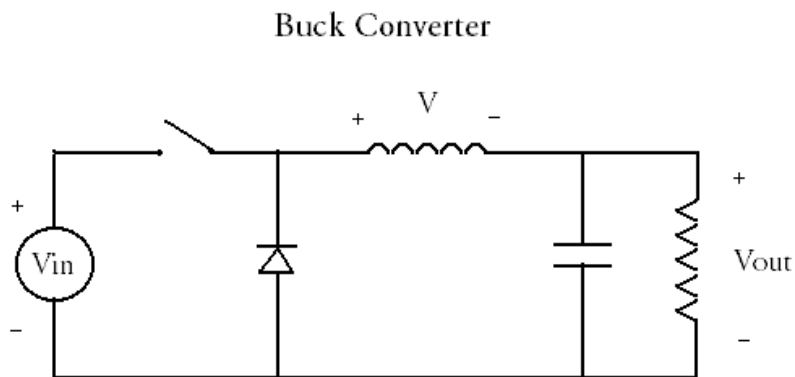
While these derivations are presented in texts as straightforward, about 3 hours of class time, two hours with a teaching assistant, many hours before paper and many many hours during the paper writing were spent attempting to derive DCM equations for output ripple voltage. The concept is straightforward, but the derivations are non-trivial.

All of the analysis in this paper utilize the constant output current model and constant output voltage model. Clearly, this is a first-order approximation to the system, since the output voltage is equal to the capacitor voltage, yet we are deriving an expression for the change in capacitor voltage. There are no component losses included in the analysis either. The main point of the paper is to reconcile the DCM output voltage ripple with the CCM output voltage ripple.

Generally,  $DT$  will be the fraction of a period where the switch in the circuit is closed, and  $T-DT$  will be the fraction of a period where the switch in the circuit is open.  $D_1T-DT$  will be the fraction of a period where the inductor current is not zero when the switch is open.

### Buck converter

A diagram of a simplified buck converter is shown in figure 1.



*Figure 1 - Buck Converter*

Diagrams of inductor current, capacitor current, and capacitor voltage are given in figure 2 for the buck converter.

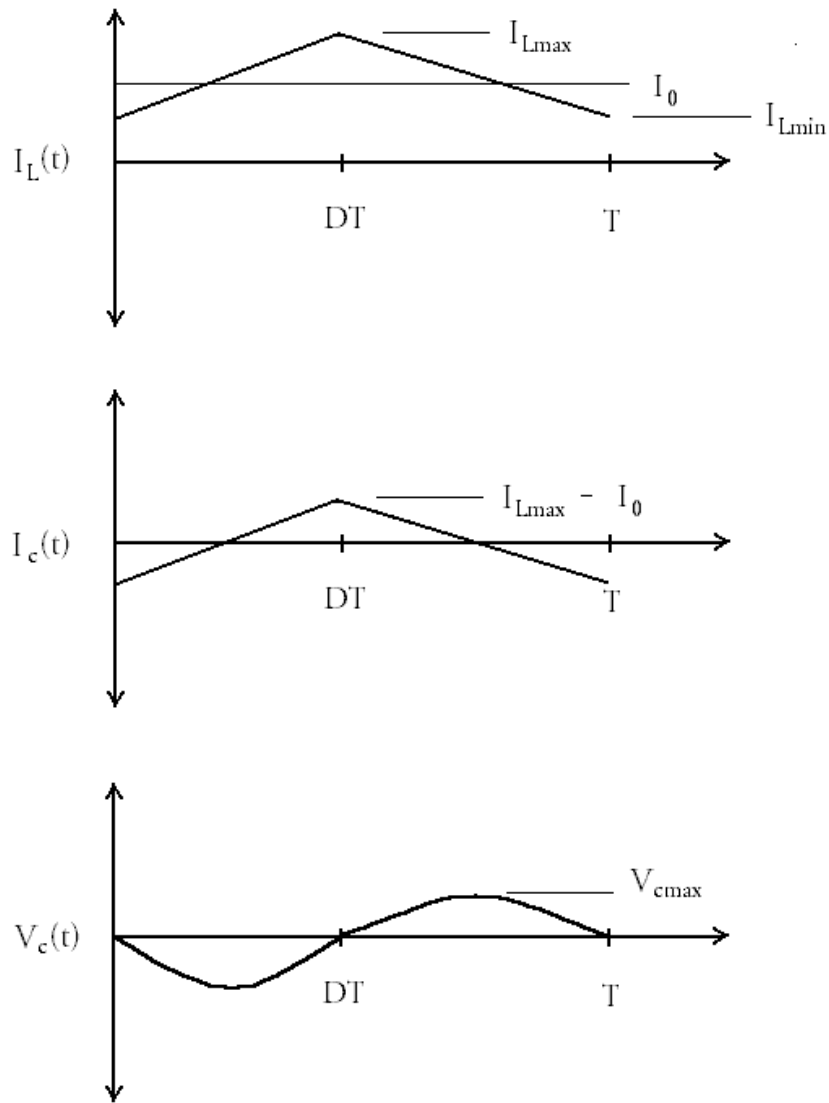


Figure 2 - inductor and capacitor current, capacitor voltage - buck CCM

For the buck converter, all of the capacitor current passes through the inductor. In mode 1, the switch is closed, and in mode 2 the switch is open. Clearly, the average output current is given by:

$$I_0 := \frac{\Delta I_L}{2} + I_{Lmir}$$

For mode 1, where  $0 \leq t \leq DT$ , and mode 2 where  $DT \leq t \leq T$  the capacitor current is given as the inductor current minus the DC component. For mode 1:

$$i_c(t) := i_L(t) - I_0$$

or

$$i_c(t) := \frac{\Delta L}{DT} \cdot t - I_0$$

For mode 2:

$$i_c(t) := \frac{-\Delta L}{(1-D) \cdot T} \cdot (t - DT) + \Delta L - I_0$$

To get the capacitor voltage, we integrate the capacitor current and divide by the capacitance.

$$v_c(t) := \frac{1}{C} \cdot \left( \frac{\Delta L}{2 \cdot DT} \cdot t^2 - I_0 \cdot t \right) + V_c(0) \quad 0 \leq t \leq DT \quad (1)$$

$$v_c(t) := \frac{1}{C} \cdot \left[ \frac{-\Delta L}{(1-D) \cdot T} \cdot \frac{(t - DT)^2}{2} + (\Delta L - I_0) \cdot t \right] + V_c(DT) \quad DT \leq t \leq T \quad (2)$$

Now, with a complete model of the capacitor voltage in both modes, we can find the change in the capacitor voltage within a switching period. Looking at the graph of the capacitor voltage in figure 2, we can see that minimum and maximum values occur halfway between each mode of operation.

Using equation 1 above, we can get the minimum capacitor voltage:

$$V_{cmin} := v_c\left(\frac{DT}{2}\right) \quad 0 \leq t \leq DT$$

$$v_c\left(\frac{DT}{2}\right) := \frac{1}{C} \cdot \left[ \frac{\Delta L}{2 \cdot DT} \cdot \left(\frac{DT}{2}\right)^2 - I_0 \cdot \frac{DT}{2} \right] + V_0$$

$$V_{cmin} := \frac{1}{C} \cdot \frac{-\Delta L \cdot DT}{8} + V_0$$

Likewise, the maximum capacitor voltage is obtained from equation 2 above:

$$V_{cmax} := v_c \left[ \frac{T \cdot (1 - D)}{2} \right] \quad DT \leq t \leq T$$

$$V_c \left[ \frac{T \cdot (1 - D)}{2} \right] = \frac{1}{C} \left[ \frac{-\Delta I_L}{(1 - D) \cdot T} \cdot \left[ \frac{T \cdot (1 - D)}{2} - DT \right]^2 + (\Delta I_L - I_0) \cdot \frac{T \cdot (1 - D)}{2} \right] + V_0$$

$$V_{cmax} := \frac{1}{C} \cdot \frac{\Delta I_L \cdot T \cdot (1 - D)}{8} + V_0$$

To find the capacitor voltage ripple (also the output voltage ripple), we need a representation for the inductor current in terms of input voltage. The output voltage terms (DC) will cancel when the minimum capacitor voltage is subtracted from the maximum capacitor voltage. Knowing that the inductor voltage is,

$$v_L := V_{in} - V_{out}$$

and the output voltage can be written as

$$V_{out} := D \cdot V_{in}$$

For a buck converter in CCM, the following integral for the inductor current can be evaluated:

$$i_L(t) := \frac{1}{L} \cdot \int_0^{DT} v_L(t) dt + I_L(0)$$

Evaluating the integral at DT to get the maximum inductor current, and at 0 to get the minimum inductor current, we have:

$$i_{Lmax} := \frac{V_{in}}{L} \cdot (1 - D) \cdot DT + I_L(0)$$

$$i_{Lmin} := I_L(0)$$

Subtracting the minimum inductor current from the maximum inductor current, the maximum

change in the inductor current can be found:

$$\Delta I_L := \frac{V_{in}}{L} \cdot (1 - D) \cdot DT$$

Using the above equation for  $\Delta I_L$ , and going back to the equations derived for  $V_{cmax}$  and  $V_{cmin}$ , the output voltage ripple can now be obtained in terms of the input voltage:

$$\Delta V_c := V_{cmax} - V_{cmin}$$

$$\Delta V_c := \left[ \frac{1}{C} \cdot \frac{\Delta I_L \cdot T \cdot (1 - D)}{8} + V_0 \right] - \left[ \frac{1}{C} \cdot \frac{-(\Delta I_L) \cdot DT}{8} + V_0 \right]$$

$$\Delta V_c := \frac{1}{C} \cdot \frac{\Delta I_L \cdot T \cdot (1 - D + D)}{8}$$

$$\Delta V_c := \frac{1}{C} \cdot \frac{\Delta I_L \cdot T}{8}$$

$$\Delta V_c := \frac{V_{in} \cdot (1 - D) \cdot D \cdot T^2}{8 \cdot L \cdot C}$$

where T is the switching period of the switch in figure 1, and D is the fraction of the period that the switch remains closed.

The buck converter operating in DCM will now be analyzed and compared to the CCM. For the DCM, the capacitor voltage ripple is best found by integrating over the positive (or negative) capacitor current. Figure 3 shows the key waveforms for the buck converter in DCM.  $D_1T$  is the fraction of a period where the inductor current reaches zero, resulting the DCM.



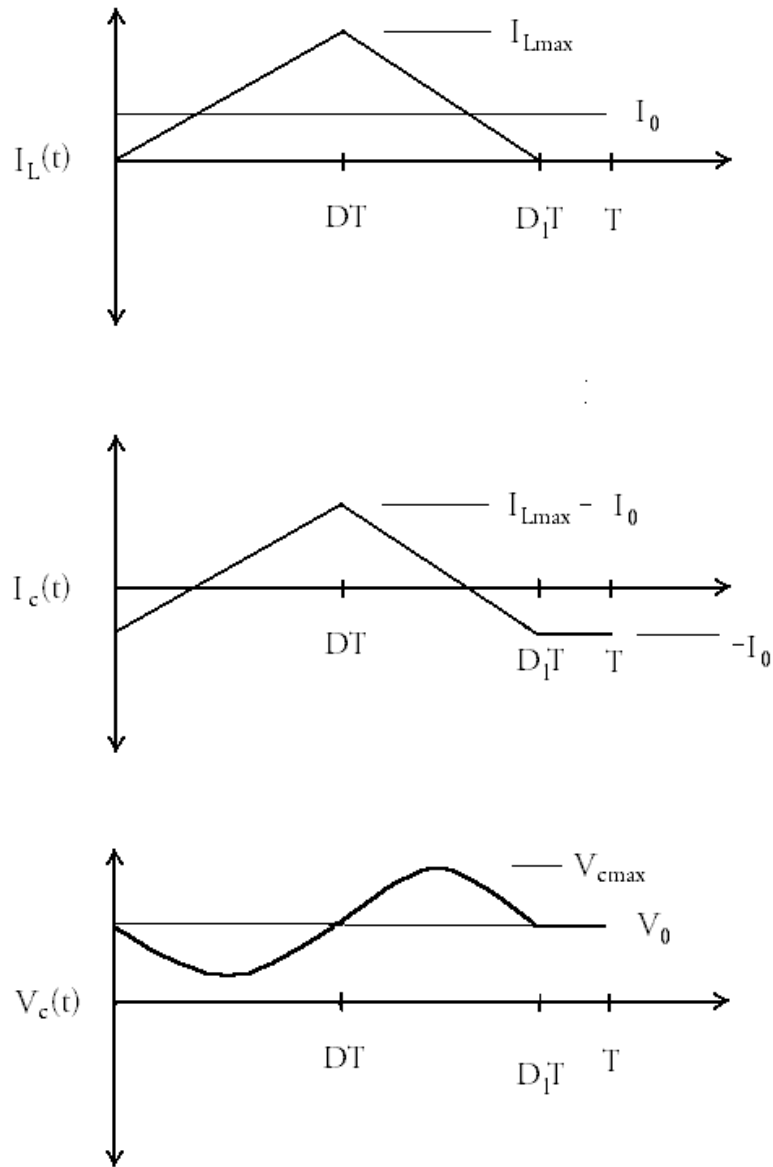


Figure 3 - Inductor and Capacitor Current, and Capacitor Voltage  
Buck DCM

Using the same analysis methods as for the CCM, the following expressions are presented for DCM:

$$V_0 := \frac{D}{D_1} \cdot V_{in}$$

$$\Delta I_L := \frac{V_{in}}{L} \left( 1 - \frac{D}{D_1} \right) \cdot DT$$

As before, all of the inductor current flows to the output, allowing the steady state output current  $I_0$  to be found in terms of  $\Delta I_L$ . The equations for the inductor current remain unchanged for mode 1 and mode 2. Mode 3, which occurs from  $D_1 T \leq t \leq T$ , has inductor current equal to zero.

$$I_0 := \frac{1}{T} \left[ \int_0^{DT} \frac{\Delta I_L}{DT} \cdot t dt + \int_{DT}^{D_1 T} \left[ \frac{-\Delta I_L}{(D_1 - D) \cdot T} \cdot (t - DT) + \Delta I \right] dt + \int_{D_1 T}^T 0 dt \right]$$

$$I_0 := \frac{\Delta I_L \cdot DT}{2} + \frac{\Delta I_L \cdot (D_1 - D) \cdot T}{2}$$

$$I_0 := \frac{D_1 \cdot \Delta I_L}{2}$$

The capacitor current in both modes remains the same. We need to use the equations to find the times  $t_1$  and  $t_2$  where the capacitor current is zero. This will allow us to integrate over the positive capacitor current to get the capacitor output voltage. Evaluating the appropriate equation for capacitor current in a given mode, and setting these equations equal to zero,  $t_1$  and  $t_2$  can be (not so easily) found in terms time parameters only. For  $t_1$ :

$$i_c(t_1) := \frac{\Delta I_L}{DT} \cdot t_1 - I_0 \quad 0 \leq t_1 \leq DT$$

$$i_c(t_1) := \frac{\Delta I_L}{DT} \cdot t_1 - \frac{D_1 \cdot \Delta I_L}{2}$$

$$t_1 := \frac{D_1 \cdot D \cdot T}{2}$$

And for  $t_2$ :

$$i_c(t_2) := \frac{-\Delta I_L}{(D_1 - D) \cdot T} \cdot (t_2 - DT) + \Delta I_L - I_0 \quad DT \leq t_2 \leq T$$

$$i_c(t_2) := \frac{-\Delta I_L}{(D_1 - D) \cdot T} \cdot (t_2 - DT) + \Delta I_L - \frac{D_1 \cdot \Delta I_L}{2}$$

$$t_2 := \left[ \left( 1 - \frac{D_1}{2} \right) \cdot (D_1 - D) \cdot T \right] + DT$$

$$t_2 := \left( D_1 - \frac{D_1^2}{2} - D + \frac{D_1 \cdot D}{2} + D \right) \cdot T$$

$$t_2 := \left( D_1 - \frac{D_1^2}{2} + \frac{D_1 \cdot D}{2} \right) \cdot T$$

The capacitor voltage is given by:

$$v_c(t) := \frac{1}{C} \cdot \int_{t_1}^{t_2} i_c(t) dt + V_c(t_1)$$

Ignoring the initial voltage, a DC component, the above integral yields  $\Delta V_C$ .

$$\Delta V_c := \frac{1}{C} \cdot \frac{1}{2} \cdot (t_2 - t_1) \cdot (\Delta I_L - I_0)$$

Substituting for  $t_1$ ,  $t_2$ ,  $I_0$ , and  $\Delta I_L$ , the output voltage ripple can be obtained in terms of  $V_{in}$ .

$$\Delta V_c := \frac{1}{2 \cdot C} \cdot \left[ \left( D_1 - \frac{D_1^2}{2} + \frac{D_1 \cdot D}{2} \right) \cdot T - \frac{D_1 \cdot D \cdot T}{2} \right] \cdot \left[ \frac{V_{in}}{L} \cdot \left( 1 - \frac{D}{D_1} \right) \cdot DT - \frac{D_1 \cdot \Delta I_L}{2} \right]$$

$$\Delta V_c := \frac{1}{2 \cdot C} \cdot \left[ \left( D_1 - \frac{D_1^2}{2} \right) \cdot T \right] \cdot \left[ \frac{V_{in}}{L} \cdot \left( 1 - \frac{D}{D_1} \right) \cdot DT \cdot \left( 1 - \frac{D_1}{2} \right) \right]$$

$$\Delta V_c := \frac{V_{in} \cdot D \cdot T^2}{2 \cdot L \cdot C} \cdot \left( D_1 - \frac{D_1^2}{2} \right) \cdot \left( 1 - \frac{D}{D_1} \right) \cdot \left( 1 - \frac{D_1}{2} \right)$$

$$\Delta V_c := \frac{V_{in} \cdot D \cdot T^2}{2 \cdot L \cdot C} \cdot \frac{1}{2} \cdot D_1 \cdot (2 - D_1) \cdot \left(1 - \frac{D}{D_1}\right) \cdot \frac{1}{2} \cdot (2 - D_1)$$

$$\Delta V_c := \frac{V_{in} \cdot D \cdot T^2}{8 \cdot L \cdot C} \cdot D_1 \cdot \frac{1}{D_1} \cdot (D_1 - D) \cdot (2 - D_1)^2$$

$$\Delta V_c := \frac{V_{in} \cdot D}{8 \cdot L \cdot C \cdot f_s} \cdot (D_1 - D) \cdot (2 - D_1)^2$$

The final equation is the equation for the output ripple voltage for the buck converter in DCM. Clearly, if  $D_1$  in the above equation is replaced by 1, the equation reduces to

$$\Delta V_c := \frac{V_{in} \cdot (1 - D) \cdot D \cdot T^2}{8 \cdot L \cdot C}$$

Which is the same result obtained from the CCM analysis. This proves the validity of our equation for  $\Delta V_C$  in DCM.

### **Boost Converter**

The analysis for CCM of the boost converter will be skipped, as the methodology is identical to the buck converter, and readily found in both references [1,2]. For DCM analysis, the following, proven equations will be used in the derivation:

$$V_0 := \frac{D_1}{D_1 - D} \cdot V_{in}$$

$$I_{in} := \frac{D_1}{D_1 - D} \cdot I_0$$

$$\Delta I_L := \frac{V_{in}}{L} \cdot DT$$

$$\Delta I_L := \left(\frac{D_1 - D}{D_1}\right) \cdot \frac{V_0}{L} \cdot DT$$

Figure 4 shows the boost converter schematic and critical waveforms for the boost converter in DCM. Again,  $D_1 T$  is the time at which the inductor current goes to zero. It should also be noted, that unlike the buck, all of the inductor current does not also flow through the

## Boost Converter

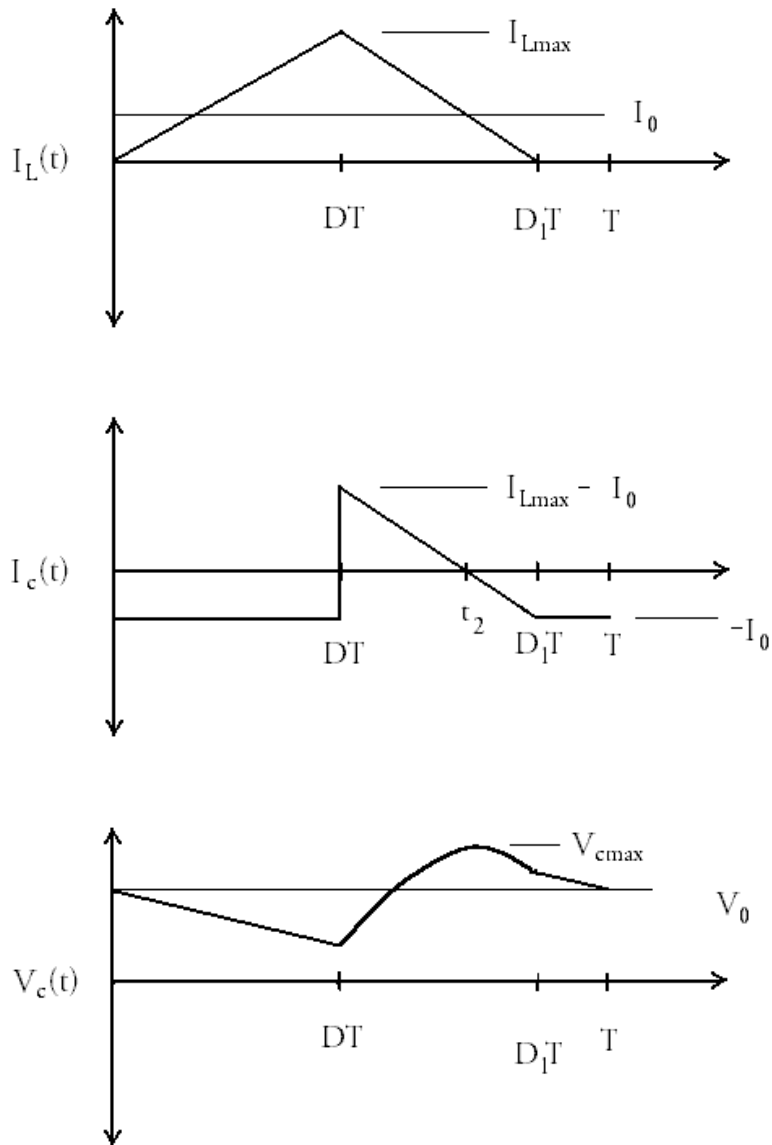
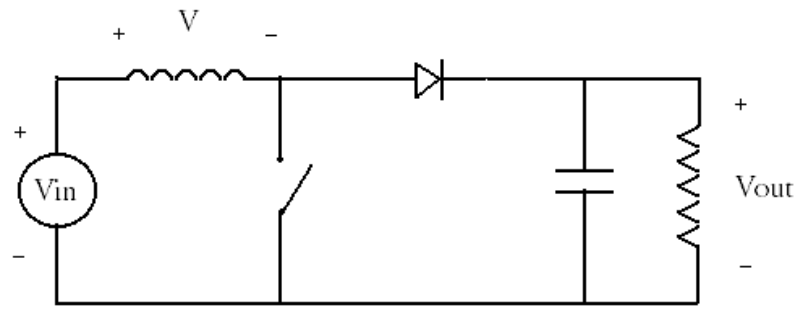


Figure 4 - Boost schematic and relevant waveforms for boost DCM

output.

From the references, the ripple voltage in CCM is given by:

$$\Delta V_c := \frac{V_0 \cdot DT}{R \cdot C}$$

and from the graph of the boost capacitor current in figure 4, it is obvious that  $t_1$ , where the positive capacitor current starts, has a value of  $DT$ .

Integrating the inductor current over one switching period will now give us the value of  $I_{in}$ , the steady state input current. Using our given equation for  $I_{in}$  in terms of  $I_{out}$ , this will again allow us to solve for the time  $t_2$ , when the capacitor current is equal to zero.

$$I_{in} := \frac{1}{T} \left[ \int_0^{DT} \frac{V_{in}}{L} \cdot dt + \int_{DT}^{D_1 \cdot T} \left[ \frac{-\Delta L}{(D_1 - D) \cdot T} \cdot (t - DT) + \Delta L \right] dt + \int_{D_1 \cdot T}^T 0 dt \right]$$

$$I_{in} := \frac{1}{T} \left[ \frac{V_{in}}{L \cdot 2} \cdot (DT)^2 + \frac{-\Delta L \cdot (D_1 - D) \cdot T}{2} + (\Delta L) \cdot T \cdot (D_1 - D) \right]$$

$$I_{in} := \frac{1}{T} \left[ \left[ \frac{V_{in}}{L \cdot 2} \cdot (DT)^2 \right] + \frac{-\Delta L \cdot (D_1 - D) \cdot T}{2} + \Delta L \cdot T \cdot (D_1 - D) \right]$$

$$I_{in} := \frac{1}{T} \left[ \frac{\Delta L}{2} \cdot DT + \frac{\Delta L \cdot (D_1 - D) \cdot T}{2} \right]$$

$$I_{in} := \frac{1}{T} \left[ \Delta L \cdot \frac{(D_1) \cdot T}{2} \right]$$

Also, manipulating the above equation,

$$I_{in} := \Delta L \cdot \left( \frac{D_1}{2} \right)$$

and

$$\Delta I_L := \frac{2 \cdot I_{in}}{D_1}$$

$$\Delta I_L := 2 \cdot \left( \frac{1}{D_1 - D} \cdot I_0 \right)$$

In mode 2, it should be obvious from the graphs in figure 4, that the capacitor current is the same as it was for mode 2 of the buck converter. Using the equation for capacitor current,  $t_2$  can again be solved for by setting the current equal to zero.

$$i_c(t_2) := \frac{-\Delta I_L}{(D_1 - D) \cdot T} \cdot (t_2 - DT) + \Delta I_L - I_0 \quad DT \leq t_2 \leq D_1 \cdot T$$

$$i_c(t_2) := \frac{-\Delta I_L}{(D_1 - D) \cdot T} \cdot (t_2 - DT) + \Delta I_L - \Delta I_L \cdot \left( \frac{D_1 - D}{2} \right)$$

and

$$t_2 := \left[ \Delta I_L - \Delta I_L \cdot \left( \frac{D_1 - D}{2} \right) \right] \cdot \frac{T(D_1 - D)}{\Delta I_L} + DT$$

$$t_2 := \left[ D_1 - D - \frac{(D_1 - D)^2}{2} \right] \cdot T + D \cdot T$$

$$t_2 := \left[ D_1 - \frac{(D_1 - D)^2}{2} \right] \cdot T$$

Again, the capacitor voltage is given by:

$$v_c(t) := \frac{1}{C} \cdot \int_{t_1}^{t_2} i_c(t) dt + V_c(t_1)$$

Once again, ignoring the initial voltage, a DC component, the above integral yields  $\Delta V_C$ .

$$\Delta V_c := \frac{1}{C} \cdot \frac{1}{2} \cdot (t_2 - t_1) \cdot (\Delta I_L - I_0)$$

Substituting for  $t_1$ ,  $t_2$ ,  $I_0$ , and  $\Delta I_L$ , the output voltage ripple can be obtained in terms of  $V_{out}$ .

$$\Delta V_c := \frac{1}{2 \cdot C} \cdot \left[ \left[ D_1 - \frac{(D_1 - D)^2}{2} \right] \cdot T - DT \right] \cdot \left( \frac{2 \cdot I_0}{D_1 - D} - I_0 \right)$$

$$\Delta V_c := \frac{1}{2 \cdot C} \cdot \left[ \left[ D_1 - \frac{(D_1 - D)^2}{2} \right] \cdot T - DT \right] \cdot \left[ \frac{V_0}{R} \cdot \left( \frac{2}{D_1 - D} - 1 \right) \right]$$

Now, hours of algebraic manipulation, equation checking, and tons of cursing will not reduce the above equation to the voltage ripple of the boost CCM given as:

$$\Delta V_c := \frac{V_0 \cdot DT}{R \cdot C}$$

Setting  $D_1$  equal to 1 in the ripple voltage equation for DCM leaves us with:

$$\Delta V_c := \frac{1}{2 \cdot C} \cdot \left[ \left[ 1 - \frac{(1 - D)^2}{2} \right] \cdot T - DT \right] \cdot \left[ \frac{V_0}{R} \cdot \left( \frac{2}{1 - D} - 1 \right) \right]$$

$$\Delta V_c := \frac{V_0 \cdot T}{2 \cdot R \cdot C} \cdot \left[ \left[ 1 - \frac{(1 - D)^2}{2} \right] - D \right] \cdot \left( \frac{2}{1 - D} - 1 \right)$$

$$\Delta V_c := \frac{V_0 \cdot T}{4 \cdot R \cdot C} \cdot (1 + D)^2$$

Why is this? The reconciliation problem is a result of the output voltage ripple voltage definition of the CCM. In my opinion, both references state the CCM output voltage ripple incorrectly, or at least incompletely. The given definition of voltage ripple only applies to the current output in figure 5, where the capacitor current in mode 2 never goes below zero. It does not apply to all cases of CCM.



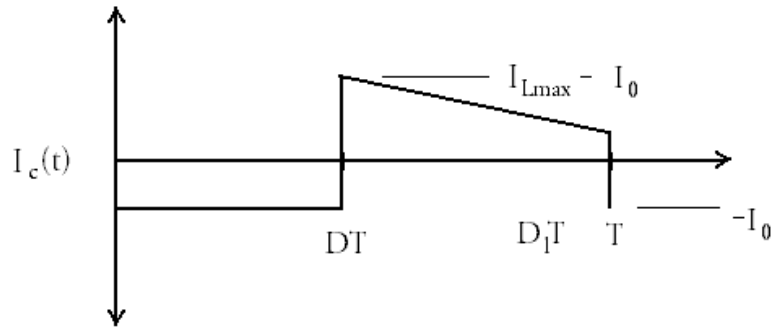


Figure 5 - Capacitor current with mode 2 current always  $> 0$

Figure 6 shows a CCM case for a boost converter where  $D_1$  is equal to 1 (boundary condition). There is area under the  $I=0$  boundary in mode 2. This area was not accounted for in the given definition for output voltage ripple for the boost converter in CCM.

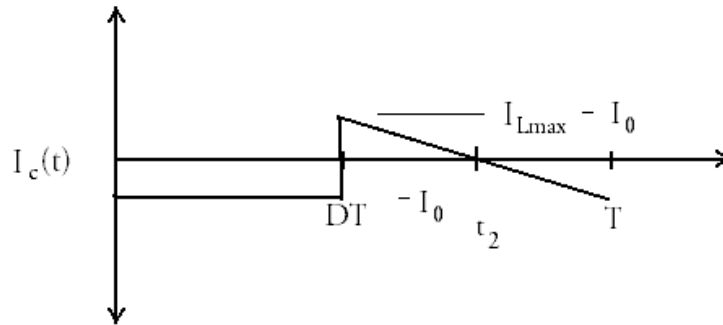


Figure 6 - Capacitor current with  $D1 = 1$  for boost converter.

Using this model to derive the output voltage ripple in CCM, and using the area under the  $I=0$  line for  $I_c(t)$ , as was done in the references to obtain the capacitor voltage ripple in CCM, we get:

$$\Delta V_c := \frac{V_0 \cdot D \cdot T}{R \cdot C} + \frac{1}{2} \cdot (T - t_2) \cdot \frac{V_0}{R \cdot C}$$

$$\Delta V_c := \frac{V_0 \cdot T}{R \cdot C} \left[ D + \frac{1}{2} \cdot \left( 1 - \frac{t_2}{T} \right) \right]$$

If we now substitute in the value for  $t_2$  that was obtained in the DCM analysis, the result is:

$$\Delta V_c := \frac{V_0 \cdot T}{R \cdot C} \left[ D + \frac{1}{2} \left[ 1 - \frac{1}{T} \cdot T \left[ 1 - \frac{(1-D)^2}{2} \right] \right] \right]$$

When the value of  $D_1$  is set to 1. The above equation further reduces to produce  $\Delta V_C$ .

$$\Delta V_c := \frac{V_0 \cdot T}{R \cdot C} \left[ D + \frac{1}{2} \left[ \frac{2}{2} - \frac{1}{2} \cdot [2 - (1-D)^2] \right] \right]$$

$$\Delta V_c := \frac{V_0 \cdot T}{4 \cdot R \cdot C} \left[ 4 \cdot D + 2 \cdot \frac{1}{2} \cdot (1-D)^2 \right]$$

$$\Delta V_c := \frac{V_0 \cdot T}{4 \cdot R \cdot C} \cdot (4 \cdot D + 1 - 2 \cdot D + D^2)$$

$$\Delta V_c := \frac{V_0 \cdot T}{4 \cdot R \cdot C} \cdot (D + 1)^2$$

As expected, this is the same result obtained when analyzing the area under the positive capacitor current in the DCM mode when  $D_1$  is set to 1.

What happens if  $-I_0 \leq I_c(T) \leq 0$ ? Modifications must be made to the DCM analysis to produce an answer in the CCM mode. Specifically, the slope of the capacitor current must be adjusted in mode 2.

$$i_c(t_2) := \frac{-\Delta I_{Lmode2}}{(1-D) \cdot T} \cdot (D_2 \cdot T - DT) + \Delta I_L - I_0$$

This is not a trivial case. These results would be expected if designing a converter around a value of L critical. Using this new equation, and setting it equal to  $-I_0$  we get a value we will call  $D_2$ .

$$-I_0 := \frac{-\Delta I_{Lmode2}}{(1-D) \cdot T} \cdot (D_2 \cdot T - DT) + \Delta I_L - I_0$$

The change in inductor current in mode 2 can be obtained from the inductor current equation for mode 2, remembering that  $V_L = V_{in} - V_0$  for a first order analysis. Solving for  $D_2$ , we obtain:

$$D_2 \cdot T := (1 - D) \cdot T \cdot \frac{\Delta L}{\Delta L_{mode2}} + D \cdot T$$

$$D_2 := (1 - D) \cdot \frac{\Delta L}{\Delta L_{mode2}} + D$$

where  $D_2$  is larger than  $T$ , and can now be used to find  $t_2$  by using the DCM equation for  $t_2$  and substituting  $D_2$  for  $D_1$  without further changes. It follows that  $t_2$  can now be used to evaluate the following integral to obtain the capacitor voltage ripple.

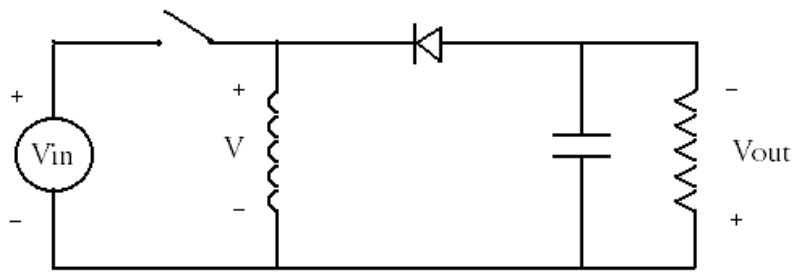
$$v_c(t) := \frac{1}{C} \int_{DT}^{t_2} i_c(t) dt + V_c(DT)$$

It is important to evaluate the initial capacitor voltage at time  $DT$  using CCM analysis, but use the equation for  $i_c(t)$  in the DCM analysis changing,  $D_1$  to  $D_2$ . However, the initial capacitor voltage, a DC value, is not required if the desired result is only the output ripple voltage. The CCM output ripple voltage is not exactly straightforward, but it will produce a good first order solution for the output (capacitor) voltage ripple for this case.

### **Buck-Boost Converter**

Well, if you ever needed some luck, it can be found with the buck-boost analysis. The buck boost converter is shown in figure 7.

## Buck-Boost Converter



*Figure 7 - Buck-Boost Converter.*

The relevant waveforms are identical to those in figure 4.

Mode 2 inductor current is just like the boost converter. Mode 1 and mode 3 capacitor current in DCM are simply  $-I_0$ . The analysis produces the same output capacitor voltage ripple as found in the boost converter.

## Conclusion

This paper has presented a step-by-step first order analysis of the buck and boost converter output ripple voltage in DCM. The buck-boost converter output ripple voltage is exactly like the boost output ripple voltage in DCM. The equations for the output ripple voltage in DCM were shown to reduce to the output ripple voltage equations in CCM. Important notes were given on the boost converter capacitor current to show what must be done for the equations to reconcile properly.

The texts advertise the process as straightforward, and indeed the *concept* is quite basic. From an educational standpoint however, it is not straightforward to ask a student to reduce the DCM output ripple voltage to the given

$$\Delta V_c := \frac{V_0 \cdot DT}{R \cdot C}$$

as it is not possible when setting  $D_1 = 1$  in the equation for DCM. How many hours does a student spend working before the student realizes the problems with the given equation pointed out in this paper? 5, 10, 20, 40, or more? You would question the integrity of those claiming to have proved the output ripple voltage in DCM does reduce to the above equation. IT DOES NOT. The information in this paper should be given up front so the obvious fact, that integrating the positive capacitor current results in the output ripple voltage for any mode of operation, can be realized and the student can move on to more difficult conceptual topics.

It should also be noted that it is rather challenging to get the equations for  $t_1$ ,  $t_2$ ,  $I_0$ , and  $\Delta I_L$  in terms of  $V_{in}$  or  $V_{out}$  correct. Special attention to these parameters is required to avoid unnecessary mistakes when deriving output ripple voltage. The graphs of the inductor and capacitor current are useful, but do not tell the whole story. The proper assumptions about the gain and order of the model must be made and remain consistent throughout the analysis.

Finally, an equation was given and all the parameters defined to obtain the output

capacitor ripple voltage for the boost and buck-boost converter in CCM for *all* cases. This equation is actually more difficult to obtain than the DCM counterpart. This paper is a must read for any student studying the basic converters in power electronics for the first time, at any level. It does indeed help the supposedly straight forward become realizable if the student is inclined to work out all of the details of the output ripple voltage for basic converters, as students should.

## **References**

- [1] Mohan, N., Undeland, T. M., Robbins, W. P., Power Electronics: Converters, Applications, and Design, Second Edition, John Wiley and Sons, Inc., New York, NY, 1995.
  
- [2] Batarseh, Issa, Power Electronic Circuits, University of Central Florida (not yet in print)